WAVELET DOMAIN IMAGE RESOLUTION ENHANCEMENT WITH DENOISING AND RESTORATION OF IMAGE HAVING ADAPTIVE EDGE PRESERVATION

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Abstract
We use wavelet domain interpolation for magnifying image without loss in clarity. Resolution of image is enhanced at different scales using wavelet methods. The image function is expanded in terms of a scaling function at a particular resolution \( j_0 \) and wavelet functions at different resolutions greater than or equal to \( j_0 \). The wavelet coefficient in multiresolution analysis are estimated using statistical models. As a result of above interpolation we obtain noticeably sharper images detecting wavelet transform modulus maxima and decomposition with total variation minimization, we obtain deblurring of images and as a result edges are distinct and sharper. Making use of periodic time invariance of wavelet transform and the wavelet threshold version of noisy signals, denoising can be affected and we could preserve the sharpness of edges.

Keywords: Multiwavelets, multiresolution analysis, wavelet transforms modulus maximum, denoising and total variation minimization.

Introduction
The imaging process results several (L) low resolution images \( y_k \), k = 1, 2, …..L and from low resolution images high resolution (1) image \( x \) is obtained. The LR images \( y_k \) and the HR image \( x \) is related as

\[ y_k = (DHF)_{k1}x_{i1} + n_k = B_kx + n_k \]  

where \( y_k \) and \( n_k \) are N x 1 vectors and \( x \) is PN x1 vector. The LR images \( y_k \) consist of n pixels and HR images \( x \) has PN pixel, \( P > 1 \) is a factor of increase of pixel. In eq(1) \( D \) is the (N x PN) down sampling matrix \( H \) is the (PN x PN) blurring matrix and \( F \) is the effect of translational(2) motion operation (PN x PN) matrix. The effect of down sampling, blurring and translation is equivalent to a single (N x PN) matrix \( B_k \), \( n_k \) is the noise matrix.

Inverting equation (1), it is possible to get the unknown super resolution image \( x \) but it is difficult to invert equation (1) because it is an ill posed problem. This is because the matrix \( B_k \) is non singular. Prior knowledge about the down sampling, blurring and translational matrix can be combined to make the problem well posed.

The blurring operation \( H \) is the overall effect of given physical conditions of imaging process. In the case of the imaging by ultraviolet radiation blurring is due to scattering and in the case of imaging by visible radiation blurring is due to diffraction effect. This sort of blurring is approximated by point spread function. This point spread function and hence the blurring operation is spatially invariant and can be represented by convolution operation with a known space invariant kernel. So the blurring operation matrix is a block circular matrix. Out of K+1 low resolution images, we assume that the initial image \( y_0 \) which refers to
the standard coordinate system, that is \( y_0 = F_0 x \) and we have \( F_0 = I \), 1-identity matrix and hence we need to estimate only k translation vectors and we consider only global translation. We assume that circular matrix \( H \) commutes with \( F \). Hence the operation in eqn (1) is split into two steps as (1) \( Z = H x \) and (2) \( y = D F Z \). So the process of determination of super resolution image \( x \) from low resolution images involves two steps

1) To obtain \( Z \) from \( y \), using the eqn (1), \( (y_k - n_k) = (D F)_{kj} z_j \). This process is called reconstruction of image.
2) \( z_j = H_{ji} x_i \). To obtain \( x \) from \( z \) is called restoration of image.

In reconstruction we include image registration and interpolation of image. Image registration furnishes the motion details of image capturing process. The motion data of the image capturing process is expressed in terms of three parameters

1) displacement \( h \) in the horizontal direction
2) displacement \( k \) in the vertical direction
3) Rotation angle \( \theta \) about z direction.

The low resolution images are superposed on a standard grid Interpolation is the process of magnifying the image many times. We propose to use wavelet method of interpolation so as to magnify image many times without loss in sharpness of image. In wavelet method, resolution is enhanced at different scales, each time incorporating higher frequency information, so as restoring the sharpness of image. For enhancement of resolution, we include higher resolution information. In this information by extrapolating the wavelet transform across scales and the wavelet transform modulus maxima captures the sharp variation of signal and their evolution across scales characterizes the local regularity of signal.

For restoration of image that is deblurring and denoising of image we apply diagonal form of wave packets. We incorporate total variation and wave packet approach for image deblurring and denoising. Further denoising near edges, directional cycle spinning is found to be very effective. This is verified by calculating PSNR of the image with directional cycle spinning.

**Wavelet analysis**

Fourier transform is the most widely used signal analysis. But Fourier transform leads information only about frequency spectrum that is, which frequencies and how much of each frequency, the signal contains. Therefore Fourier transform method fails to analyze highly non stationary noise and a periodic signals. Those signals can be analyzed by using local analysis method. Wavelet method\(^{(3,4)}\) is a powerful tool for such a study. Wavelets supply information about both time and frequency although these parameters cannot be precisely determined due to Heisenberg’s uncertainty principle that is \( \Delta t \Delta f \leq 1/4\pi \). The significance of wavelet analysis is that it reveals signal aspect that the other analysis techniques miss, such as trends, breakdown points, discontinuities. Wavelets are small waves of very small time extent and with a very small frequency width. The energy is concentrated in time.

The following are three general characteristics of wave system

1) Wavelet system\(^{(4,5)}\) is a time dimensional expansion set. The image function \( f(t) \) is expanded as

\[
F(t) = \sum_{j} \sum_{k} a_{jk} \varphi_{jk}(t)
\]

where \( a_{jk} \) is wave coefficient.

2) The wavelet expansion is a time frequency localization of the signal. This means that most of the energy of the signal is represented by a few expansion coefficients.

3) The wavelet coefficients can be calculated efficiently by using the formula \( a_{jk} = \int \varphi_{jk}^* f(t) dt \). So wavelet transforms (set of wavelet coefficients) can be calculated with \( O(N) \) operations. More general wavelet transform require \( O(N\log N) \) operations.

There are three additional characteristics applicable to wavelets.

1) All, so called first generation wavelets are generated from a single scaling function by scaling and translation. This two dimensional
parameterization is achieved from the relation. 
\[ \varphi_{jk}(t) = 2^{j/2} \psi(2^j t - k), \quad j,k \in \mathbb{Z} \]
where \( \mathbb{Z} \) the set of all integers and the factor \( 2^{j/2} \) maintains constant norm. The time or space location is fixed by \( k \) and the frequency scale by \( j \).

2) Almost all useful wavelet systems satisfy the multi resolution condition. This means that if a set of signals are represented by a weighted sum of \( \varphi(t-k) \), then a larger set can be represented by a weighted sum of \( \varphi(2t-k) \). In other words, if the basic expansions signals, are made half as wide and translated in steps half as wide, they will represent a larger class of signals or a better approximation of the signal.

3) The lower resolution coefficients can be calculated from the higher resolution coefficient by tree structured algorithms called filter bank.

In multi resolution formulation, we need two closely related basic functions. In addition to the wavelet function, we need another basic function called scaling function. So the image function can be expanded as

\[ f(t) = \sum_k c_{j,0k} \varphi_{j,0k} + \sum_{k=\infty}^{0} \sum_{j=0}^{\infty} d_{j,k} \varphi_{jk} \]

where \( \varphi_{j0k} = 2^{j/2} \varphi(2^j t) \), \( \psi_{jk} = 2^{j/2} \varphi(2^j t - k) \),

\[ c_{j0k} = \int \varphi_{j0k}^* f(t) \, dt \] and \( d_{j,k} = \int \psi_{jk}^* f(t) \, dt \)

The desirable properties of scaling functions and wave functions are orthonormality, interpolation, compact support and approximation power. It is practically difficult to construct multiresolution wavelet satisfying the above requirements. One possible way out is multicomponent wavelets. These multiwavelets are constructed satisfying the refining equations of the form

\[ \varphi(x) = \sum_{k} A_k \varphi(2x-t) \]

where \( (A_k) \) are real (rnxr) matrices by these r-scaling vectors, we can construct r- multwavelets.

The resolution can be enhanced effectively if done at different scales of resolution. The multi resolution analysis should satisfy the following nesting property

1) \( V_0 \subset V_1 \subset \ldots \subset V_{\infty} \)
2) \( V_j \subset V_{j+1} \) for all \( j \) for all \( j \in \mathbb{Z} \)
3) \( V_{-\infty} = \{ 0 \} \) and \( V_{\infty} = \{ L^2 \} \) with requirements.

The space with high resolution contain those with low resolution also. It should satisfy the scaling condition

\[ f(t) \in v_j \iff f(2t) \in v_{j+1} \]

\( V_j \) is spanned by scaling function \( \varphi(2^j t-k) \). If we define wavelet spanned subspace \( W_0 \), then \( V_1 = V_0 + W_0 \) and generalizing the definition, we get \( V_j = V_{j-1} + W_j \) for all \( j \geq 1 \). If we generalize further \( V_j = V_{j-1} + W_j \). Therefore \( L^2 = V_0 \oplus W_0 \oplus V_1 \oplus W_1 \oplus \ldots \)

Wavelet expansion and wavelet transform are very effective in analyzing images. This is because

1) The size of wavelet expansion coefficient \( c_{jk} \) and \( d_{jk} \) drop off rapidly with \( j \) and \( k \) for large class of image function. These wavelets are effective in image compression or reconstruction and denoising.

2) The wavelet expansion allows a more local description and separation of image characteristic. A wavelet expansion coefficient represents a component that is itself local, also it allows a separation of components that overlap in both time and frequency.

3) Wavelets are adjustable and adaptable and can be designed to fit individual application.

4) The generalization of wavelet and calculation of wavelet is well matched to digital imaging.

Interpolation by wavelet techniques

High resolution image is obtained from several low resolution images. When a series of low resolution images are taken in a short interval of time, there is a relative displacement between the images owing to a small camera motion or motion of the object which is being photographed. The motion is described in terms of three parameters namely horizontal shift \( h \), vertical shift \( v \) and rotation denoted by angle of rotation \( \theta \) about \( Z \) axis. The resulting image, taking into account motion details can be expressed as

\[ F(x', y') = f(x \cos \theta - y \sin \theta + h, y \cos \theta + x \sin \theta + k). \]

Expressing

\[
\sin \theta = \theta - \frac{\theta^3}{3} \\
\cos \theta = 1 - \frac{\theta^2}{2!} + \ldots
\]

\[
f(x', y') = f(x, y) + (h - y \theta - x \theta^2/2)f_x(x, y) + (k + x \theta + y \theta^2/2)f_y(x, y) + \ldots
\]

The error function is denoted as \( E = f(x', y') - f(x, y) \).

Requiring the mean square value of error to be minimum, motion parameters can be calculated and the relative displacements of under sampled LR images can be estimated with sub pixel accuracy.

We combine these LR images in a high resolution grid.

Interpolation and hence super resolution image from low resolution image can be obtained by wavelet method. Low resolution images are first digitalized. The image function is decomposed into components at different scales or resolutions. The advantages of this method is that signal trends at different scales can be isolated and studied. The global trends can be examined at coarse scale using scaling functions whereas local variations are studied at higher scales of resolution by wavelets.

The image function \( f(t) \) is expanded as

\[
f(t) = \sum_{k \in \mathbb{C}} C_{j_0, k} \phi_{j_0, k} + \sum_{j > j_0} \sum_{k} d_{j, k} \psi_{j, k}
\]

and the expansion coefficient \( C_{j, k} = \int f(t) \phi_{j_0, k}^* \, dt \) and \( d_{j, k} = \int f(t) \psi_{j_0, k}^* \, dt \) Using these estimates, we interpolate function values at the HR grid points.

We consider the case of non uniform 1D signals and if we have to compute \( M \) uniformly distributed values say at \( t = 0, 1, 2, \ldots, M-1 \) and we make use of \( p \) non uniformly sampled data points of \( f(t) \) at \( t = t_0, t_1, \ldots, t_{p-1} \) so that \( 0 < t_s < M \). We take unit time spacing grid to be resolution level \( V_0 \). By repeated application, we decompose \( V_0 \) as \( V_0 = V_1 + \sum_{j} W_j \), where \( j \leq -1 \).

Substituting the values of sampled data, we get a set of \( p \) linear equation. The desired value of \( f(t) \) at the HR grid point can be computed using the estimated coefficients.

The ideal interpolation of a point \((x, y)\) should be 1-D interpolation in the x direction first by four one dimensional interpolation and they should be used for the final 1-D interpolation along y direction. The image is decomposed into four sub regions

1. Approximate
2. Horizontal
3. Vertical
4. Diagonal components

The diagonal components are again decomposed into two diagonal directions SW-NE and NW-SE. Each of the four components is interpolated separately. Wavelets are again used to convert these subspace back to image domain. In terms of frequency, the approximate component has low horizontal and vertical frequency components. The horizontal component has low horizontal and high vertical frequency components. The vertical component has low vertical and high horizontal frequency component. The diagonal component has low diagonal high horizontal and vertical frequency components. Each of these components are up sampled independently using appropriate kernel. As a result of interpolation of wavelet method, we get high resolution images with well defined edges and textures.

The wavelet coefficients can be evaluated by modeling the wavelet coefficients as nodes in a Markov tree with a certain hidden state. In Markov process the transition probability depends only on current state. On a hidden Markov tree model, we do not know exactly the state of process. The algorithm takes as input the wavelet coefficients and produces the state transition probabilities. The mean and variance of the distribution is reckoned with reference to a standard reference set. For evaluating coefficients at finer scales we make use of probability distribution.

\[
P(w^k_c) = \frac{1}{\sqrt{2\pi} \sigma_{c,m}^k} e^{-\frac{(w^k_c - \mu_{c,m}^k)^2}{2\sigma_{c,m}^k}}
\]
where $w_c^k$ are the wavelet coefficient at scale $k$ which we want to evaluate (The variance $\sigma_{c,m}^k$ and mean $\mu_{c,m}^k$ are obtained from the training set. Using the above probability distribution equations we determine the wavelet coefficient at different scale of resolution. The wavelet coefficient is the measure of information about the image at that scale of resolution)

**Restoration of image-resolution enhancement as a result of deblurring and denoising operation**

For restoration of image, we propose following three methods

1. Wavelet transform extrema extrapolation
2. Cycle spinning with edge modeling
3. Simultaneous decomposition, deblurring and denoising by means of wavelet –total variation method

a) Resolution enhancement of image by wavelet transform extrema extrapolation

During interpolation, we magnify image many times and it may result loss in sharpness of image. Using wavelet based method, we estimate the higher resolution information needed so that the sharpness of image is restored.

The available image function $f$ is assumed to be obtained from the high resolution signal $f_0$. It is required to recover $f_0$. To achieve this, the image function $f$ is low pass filtered and then down sampled by a factor of 2. Denote the low pass filter by $H_0(z)$ and a high pass filter by $H_1(z)$. The analysis filter $H_0(z)$ and $H_1(z)$ together with the synthesis filters $G_0(z)$ and $G_1(z)$ constitute a perfect reconstruction non sampled filter bank and perfect reconstruction condition is $H_0(z)G_0(z) + H_1(z)G_1(z) = 1$.

For reconstruction of the high resolution image signal we need know its high pass component $g_s$ and low pass component $f_s$. But we know only $f$, the down sampled version of $f_0$. By interpolating $f$ using wavelet method, followed by high pass filtering to deblur the resulting signal $f_s$ is recovered. By our proposed algorithm, high pass component $g_s$ is found by using wavelet transform extrema extrapolation analysing the available signal $f$. Then the high frequency component $g_s$ is combined with the estimate of $f_s$, and we get reconstructed version of high frequency signal.

It is obvious that the local extrema of wavelet transform are induced by similarities in the signal. Due to these singularities wavelet coefficients at higher scales of resolutions are dominant and the extreme values corresponding to these singularities are given $w_i h(x_n^n) = K_n 2^{j\alpha_n}$.

where $w_i h$ is the wavelet transform of the input signals at resolution $j$, $x_n$ is the location of local extrema at scale $j$ corresponding to $n^{th}$ singularity, $\alpha_n$ is the Lipschitz singularity of $f$ at singular point and $K_n$ is a constant. The wavelet transform of $f$ is the decimated version by a factor 2. We extrapolate the higher scale signal $g$ from the evolution of local extrema across coarser scale. Signal enhancement is made, if we satisfy the following constraints:

1. The waveform $\left( ^\wedge f_s, ^\wedge g_s \right)$ where $^\wedge f_s, ^\wedge g_s$ are the initial estimate of $f_s$ and $g_s$ must be in the subspace $\nu$.
2. The downward version $^\wedge f_s$ is equal to $f$, the original signal.
3. The local extrema $^\wedge g_s$ should reflect the sharp variation in $f_0$, their values and locations are determined by singularities in $f_0$.

For extrapolation, we choose important singularities and corresponding extrema across scales are noted. Since the highest scale of resolution is more sensitive to noise, selection of the extrema is done at second scale. For analyzing 2D problem, we treat two 1D coordinates separately and for wavelet transform, data is filtered by separate 2D filter bank.

To obtain a test image, the original 512 X 512 image is low pass filtered subsampled by 2 and the process is repeated to obtain 128 X 128. From this 64 X 64 sub image is extracted and this is available for interpolation. Low pass filter is used to obtain the test image as a seperable 2D filter ie

\[ H(w_1,w_2) = H(w_1)H(w_2) \]

The signal extrema is determined and high frequency components \( g_s \) is deduced. Moreover low pass components \( f_s \) is also determined signal enhancement is made as \( g_s \) and \( f_s \), so as to get high frequency component \( g_s \) and low pass component \( f_s \). Combining \( f_s \) and \( g_s \) we can reconstruct high resolution image with sharp and clear edges.

b) Image Resolution Enhancement in Wavelet Domain Using Cycle Spinning and Edge Modeling

Recently several wavelet domain methodologies are developed to enhance resolution by overcoming the drawbacks of classical interpolation methods. A common feature of this algorithm is the assumption that low resolution image to be enhanced is the low pass filtered sub band of high resolution image which has been subjected to a decimated wavelet transform. A trivial approach is to reconstruct the approximation to HR image by filling the unknowns, so called detail sub bands with zeros followed by the application of inverse wavelet transform. But this method is not very efficient. So more sophisticated methods are used to estimate the unknown detail wavelet coefficient, so as to improve sharpness.

Edges are identified by an edge detection algorithms. The low pass sub band is the result of variable separable method first horizontal and then vertical low pass wavelet filtering. The unavailability of high frequency spatial information normally residing in the unknown sub bands results in blurring and ringing around sharp edges to describe edge evolution as it undergoes low pass filtering, edges are approximated by a Gaussian step function.

\[ S(x) = h(x,b,c) * g(x,w) \]

\( c \) : Edge contrast, \( b \) : Edge minimum, \( * \) : Convolution, Parameters denotes the width of edge. In the HR image, the surviving edge in the unavailable image would be represented by

\[ s_e(x) = s(x,b,c,w) + q_w(x) \]

\( s_e(x) \) being edge widening factor \( q_w \) refers to residual degradation such as ringing. The edge distortion can be obtained by establishing a correspondence between available LR image and a training set of HR image.

Our algorithm for resolution enhancement consist of two steps

1) Wavelet domain zero padding
2) Cycle spinning

The decimated wavelet transform is not shift variant and as a result, there is a distortion of wavelet coefficient. This will be visible in ringing in neighborhood of discontinuity. Cycle spinning is an effective method against ringing. Cycle spinning will reduce ringing when used for denoising and increase perceptual quality. Cycle spinning tries to approximate shift invariant statistics by averaging out cyclostationarity.

For that

1) a number of LR images are generated from \( y_0 \) by spatial shifting
2) Wavelet transform is affected \( x_{ij} \)
3) Discard high frequency coefficient i.e.

\[ x_{ij} = D W S_{ij} y_0 \]

\( D \) represents discarding HF coefficients W-Wavelet transform \( S_{ij} \) shift operator applying horizontal and vertical shift. Then apply wavelet domain zero padding. The intermediate HR images are realized and average to find HR reconstruction images. High frequency information results in larger values of width of edges. Application of cycle spinning widens edge function and we readjust edge width so as to improve the quality of edges.

It is well known that decimated wavelet transform is not shift invariant and as a result inaccurate representation of wavelet coefficient introduce cyclostationarity and manifest itself as ringing in the neighboring discontinuities. Cycle spinning minimize ringing. Cycle spinning consists of

1) Forward translation wavelet denoising and backward translation wavelet denoising. This introduce wavelet domain zero padding and find the
inverse wavelet transform. The operation is represented as
\[ \tilde{y}_0 = W^{-1} \begin{bmatrix} x & \theta_{mn} \\ \theta_{mn} & \theta_{mn} \end{bmatrix} \]

In conventional cycle spinning methods, all possible shifts about local neighborhoods are used. It is noted that ringing occurs not only in vicinity of strong edges but they are strongly correlated with the orientation of edge. For an edge of a given orientation ringing is more pronounced in the normal direction. This observation suggest that CS should predominantly applied across edges in the case of horizontal edges, CS is applied in vertical direction and in the case of vertical edges CS is applied horizontally. So we propose directional cycle spinning and our method consists of:

a) Generating LR images by low pass filtering followed by down sampling
b) Then we get approximate HR image using wavelet domain Zero padding followed CS applied at different orientation, non directional, horizontal, vertical or diagonal.

Resolution enhancement\(^{(9,10,11)}\) Simultaneous decomposition, deblurring and denoising using total variation minimization of wavelets

Image function is decomposed into:
1. Cartoon component which is piecewise smooth with possibly abrupt edges and contours
2. Oscillating or texture part. This parts fill in the smooth regions in the cartoon parts with oscillating feature. The decomposition can be represented as

\[ f = u + v, \]

\[ u \text{- Cartoon part, } v \text{- Oscillation part. } u \text{ -which contain main feature of the image can be represented as a function of bounded variation, in } SBV(\Omega), \text{ SBV denotes special functions of bounded}^{(9,14)} \text{ variation, allowing discontinuities and hence sharp edges and contours in the image. The residual part } v \text{ which is equal to } f - u, \text{ by a function in } L^2(\Omega) \text{ with bounded } H^{-1} \text{ norm.} \]

To restore true image from the observed image, we need to find \( u \), we have to solve the inverse problem. One of the most well known technique for the same is the total energy minimisation and regularization first proposed by L. Rudin, S. Osher\(^{(15,16)}\) and E. Fatemi. The decomposition of \( f \) into \( u \) and \( v \) is obtained by solving \( G_p(u, g_1, g_2) \) where

\[ G_p(u, g_1, g_2) = \int_\Omega |\nabla u| + \lambda \| f - u + \text{div } g \|^2 + \mu \| g \| \text{ with } f \in L^2(\Omega), \Omega \subset \mathbb{R}^2 \text{ and } v = \text{div } g \]

The first term refers total variation of \( u \). If \( u \in L_{-1}, |\nabla u| \) is a finite measure in \( \Omega \) and \( u \in BV(\Omega) \). This space allows discontinuities and hence edges and contours appear in \( u \). The second term \( \lambda \| f - u + \text{div } g \| \) represents the restoration discrepancy. The third term \( \mu \| g \| \) approximate the sum of the space of oscillating function. In fact \( v \)-penalty term can expressed as

\[ \| g \|_{L^2(\Omega)} = \int_\Omega |(\nabla \Delta)^{-1} u|^2 = \| u \|_{H^{-1}(\Omega)}. \]

Hence variational problem simplifies to solve the equation

\[ \inf_{u, v} G_2(u, v) \] where

\[ G_2(u, v) = \int_\Omega |\nabla u| + \lambda \| f - (u + v) \|^2_{L^2(\Omega)} + \mu \| g \|_{H^{-1}(\Omega)} \]

Instead of solving the equation by finite difference method, we suggest wavelet based techniques. In order to solve the above problem by wavelet technique we need solve the expression

\[ \inf_{u, v} \{ \sum_{\lambda \in J} (|f_\lambda - (u_\lambda + v_\lambda)|^2 + r \lambda^2 |\langle V_\lambda \rangle|^2 + 2|U_\lambda|^2) \} \]

Where \( J = \{ \lambda = i, j, k, i \in J, j \in z; i=1,2,3 \} \)

Index set used. The minimization of the above equation is straight forward, since it decouples into one dimensional minimizations

**Experimental results**

In this section, we discuss the results of our experimental study of the effects of our algorithms on resolution enhancement. A comparative analysis
of three different wavelet techniques on image restoration namely
(1) Wavelet transform extrema
(2) Denoising using total variation minimization of wavelets
(3) Cycle spinning with edge modeling, is made. The result is shown in the tabular column

Table 1: Techniques used

<table>
<thead>
<tr>
<th>Technique used</th>
<th>PSNR Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelet based interpolation (resolution 864x1152)</td>
<td>94.16 dB</td>
</tr>
<tr>
<td>Wavelet based interpolation with wavelet transform extrema for restoration of image (resolution 864x1152)</td>
<td>94.72 dB</td>
</tr>
<tr>
<td>Wavelet based interpolation with total variation minimization for restoration of image (resolution 864x1152)</td>
<td>95.21 dB</td>
</tr>
<tr>
<td>Wavelet based interpolation with cycle spinning and edge modeling for restoration of image</td>
<td>95.5181 dB</td>
</tr>
</tbody>
</table>

Conclusion

In this paper we have included algorithms for restoration of image using
1) Wavelet transform extrema
2) Decomposition of image function into cartoon part and oscillating part and extracting cartoon part from the image function
3) Cycle spinning with edge modeling. It is found that by these methods of restoration of image we get high PSNR value and resolution is enhanced

References


